Communication for maths

Term 2, week 11: The presentation of series and sequences

Describing sequences and series

Terminology

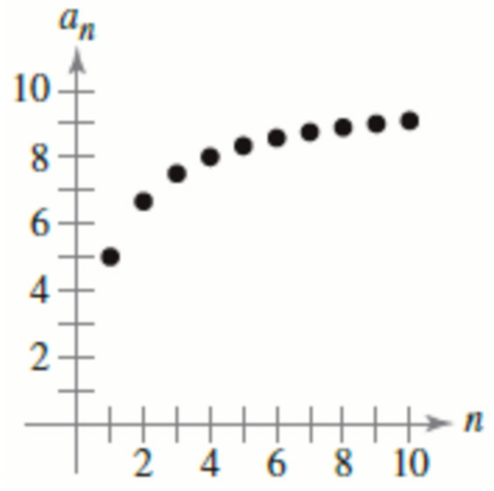
- Below is some appropriate terminology which can be used to describe sequences.

Positive	Negative	Increasing	Decreasing
Monotonic	Constant	Periodic	(Un)Bounded
Term	Repeating	Finite	Infinite
Convergent	Divergent	Limit	Oscillating

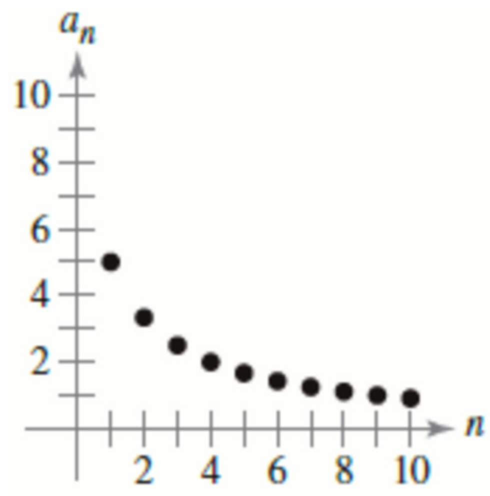
Describing sequences and series

- Using the above terminology, as well as any other appropriate terminology, describe the behaviour of the sequences in the following diagrams.

a)



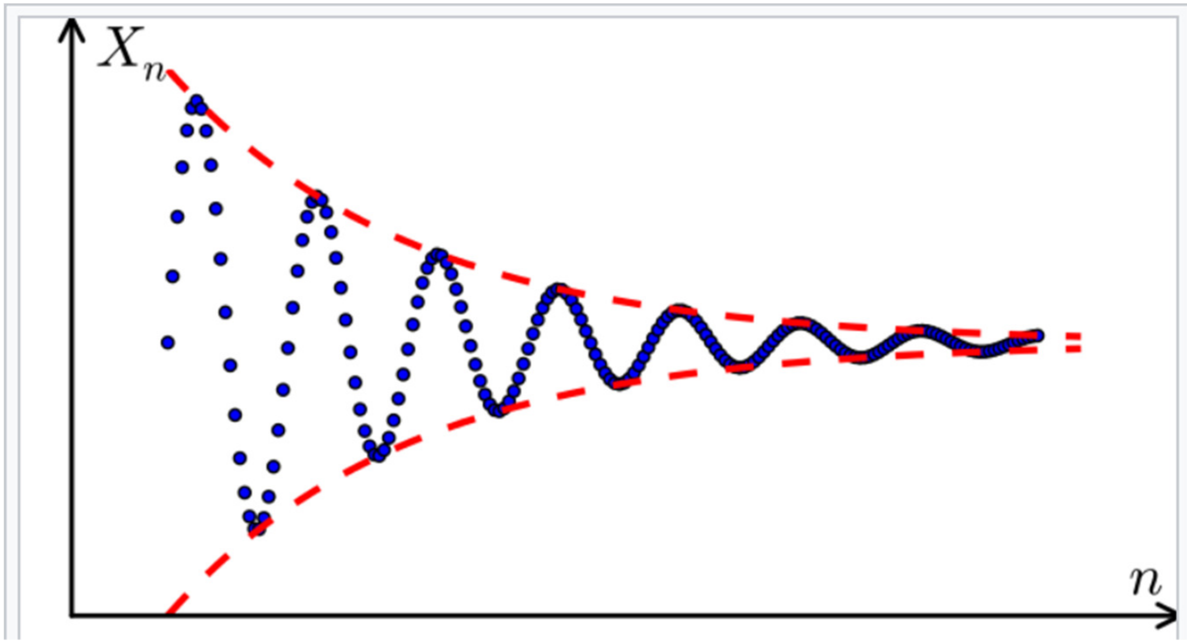
b)



Describing sequences and series

- Using the above terminology, as well as any other appropriate terminology, describe the behaviour of the sequences in the following diagrams.

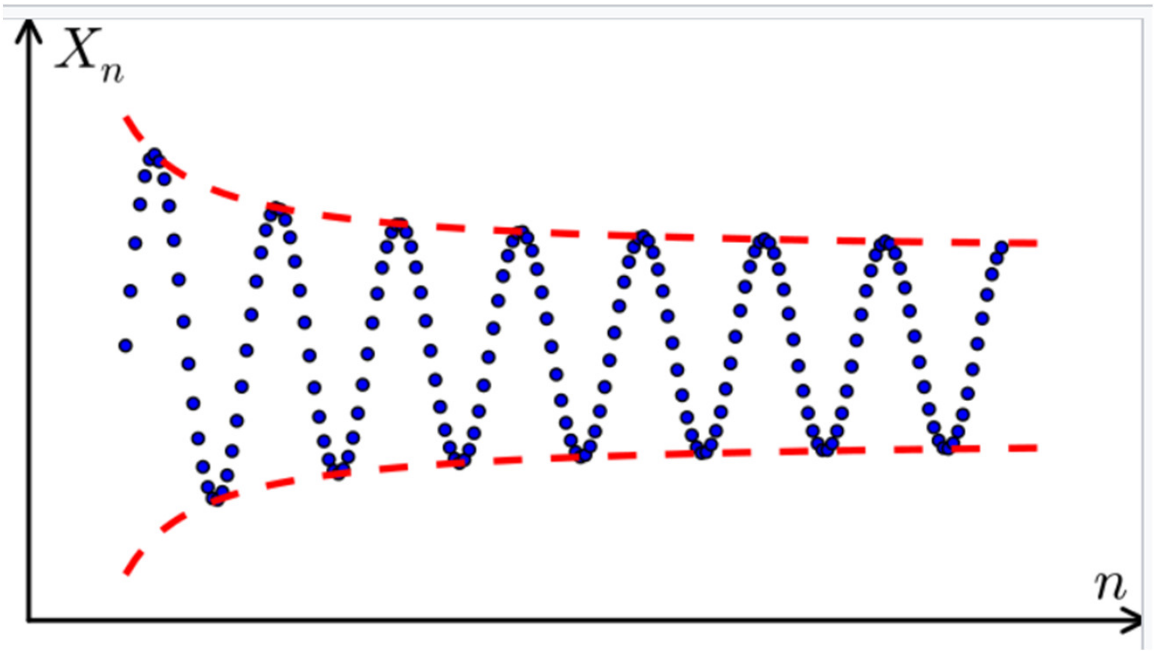
c)



Describing sequences and series

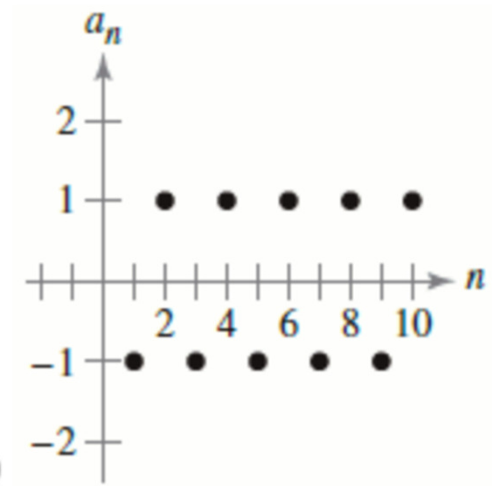
- Using the above terminology, as well as any other appropriate terminology, describe the behaviour of the sequences in the following diagrams.

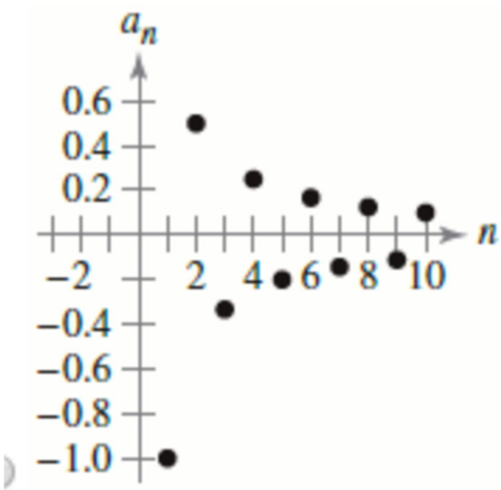
d)



Describing sequences and series

- Using the above terminology, as well as any other appropriate terminology, describe the behaviour of the sequences in the following diagrams.

e) 

f) 

Describing sequences and series

Notation

- One primary notation for stating sequences is a pair of brackets: ( ).
- This is not be confused with the same notation used for specifying a coordinate: (x, y).
- Context will tell us whether or not we are referring to a sequence or to a coordinate.

Describing sequences and series

Notation

- The most general way of writing a sequence is

$$\{a_k\}$$

where $k = 1, \ldots, n$ (or $k = 1, \ldots, \infty$)

or

$$\{a_k\}_{k=1}^{n}$$

- We can also list the individual elements:

$$\{a_1, a_2, a_3, \ldots, a_n\}$$

Describing sequences and series

Example

- We want to write the correct symbolism or mathematical expression for the following descriptions:
  a) A sequence;
  b) A sequence of integers;
  c) A sequence of positive integers x;
  d) A sequence of positive integers x such that $1 \leq x \leq 3$

Describing sequences and series

Answers: See lesson

a) A sequence:

b) A sequence of integers:

c) A sequence of positive integers x:

d) A sequence of positive integers x such that $1 \leq x \leq 3$:

Describing sequences and series

Exercise

- Write the correct mathematical expression for the following descriptions:
  1. A bounded sequence;
  2. An increasing sequence;
  3. A bounded decreasing sequence of rational numbers
  4. A bounded increasing sequence of real numbers;

Describing sequences and series

Exercise

- Write the correct mathematical expression for the following descriptions:

  5) An alternating sequence of real numbers;

  6) A sequence of positive integers with a repeating decimal part;

Describing sequences and series

Exercise

- Write mathematical expressions for the following description:

An infinite sequence of binomials

with integer coefficients
with unbounded coefficients
with increasing degree
whose leading term alternates in sign.

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Appendix

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Describing sequences and series

- Go back to Ramesh's def of increasing and decreasing functions, and write maths statements the following

Exercise 4.4. Consider the following implications, where f is a real function.

1. *If f is decreasing, then $-f$ is increasing.*
2. *If f is decreasing, then $|f|$ is increasing.*
3. *If $|f|$ is increasing, then f is monotonic.*

Describing sequences and series

Exercise

- Write mathematical expressions for the following description:

1. The sequences (a_k) and (b_k) are distinct.
2. The sequence (a_k) is eventually constant.
3. The sequence (a_k) is not periodic.
4. The sequence (a_k) is eventually periodic.
5. The sequence (a_k) has infinitely many negative terms.
6. Eventually, all terms of the sequence (a_k) become negative.
7. The terms of the sequence (a_k) get arbitrarily close to zero.
8. Each term of the sequence (a_k) appears infinitely often.
9. Each term of the sequence (a_k) appears at least twice.
10. Each natural number appears infinitely often in the sequence (a_k).

Describing sequences and series

Exercise

- Explain clearly and succinctly, using any and all appropriate mathematical terminology:
  - How do I …?*(*ask Qs about series*)*

1. How do I divide two fractions?
2. I have a positive integer. How do I check if it's prime?
3. I have a positive integer. How do I check if it's a cube?
4. I have two vectors on the plane. How do I check if they are linearly independent?
5. I have a cartesian equation of a circle, and a point. How do I check if the point lies inside the circle?
6. I have two lines in three-dimensional space. How do I check if they intersect?